



Heterogeneity in Models

Thinking about differences in populations

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What we've seen

- The fundamentals of compartmental (SIR) models
- Adding complexity with demography and seasonality
 - Better represents long-term infection patterns



What's the key assumption

All individuals are the same



How are people different?

- Age
- Gender
- Race
- SES
- Risk tolerance
- Number of contacts (through work and personal lives)



What is the goal of a (scenario) model?

Our goals will dictate what is important

Sometimes we can just use the basic SIR with demography ...

Other times we need to model the differences in individuals.



When do we need heterogeneity?

When we are designing targeted interventions

- IV drug-users are at elevated risk of being infected **and** infecting others
- Is it more effective to vaccinate younger individuals vs older during influenza season?
- Designing messaging campaigns to reduce STD transmission

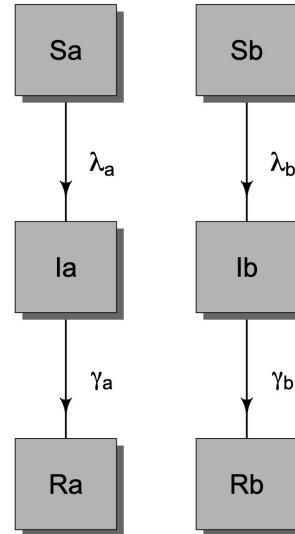
Modeling differences



How do we model differences?

Imagine we have 2 groups: 1
high-risk and 1 low-risk

We could build 2 separate
models

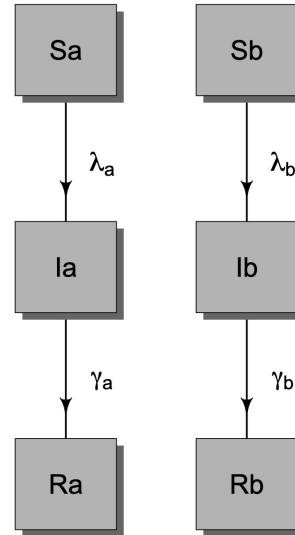


How do we model differences?

Doesn't account for interactions between the groups:

High risk group will **increase** transmission in low risk group

Low risk group will **decrease** transmission in high risk group





How do we model differences?

Can represent interactions using a **matrix**

$$\begin{pmatrix} \lambda_a \\ \lambda_b \end{pmatrix} = \begin{pmatrix} \beta_{a,a} & \beta_{a,b} \\ \beta_{b,a} & \beta_{b,b} \end{pmatrix} \begin{pmatrix} I_a \\ I_b \end{pmatrix}$$



How do we model differences?

a indicates the group **being infected**

b indicates the **infectious group**

$$\begin{pmatrix} \beta_{a,a} & \beta_{a,b} \\ \beta_{b,a} & \beta_{b,b} \end{pmatrix}$$



How do we model differences?

The diagonal represents **within-group** interactions

The off-diagonal represents **between-group** interactions

$$\begin{pmatrix} \beta_{a,a} & \beta_{a,b} \\ \beta_{b,a} & \beta_{b,b} \end{pmatrix}$$



How do we model differences?

What happens when all values are the same?

Equivalent to a 1-compartment model

$$\begin{pmatrix} \beta_{a,a} & \beta_{a,b} \\ \beta_{b,a} & \beta_{b,b} \end{pmatrix}$$

What happens when the off diagonals = 0?

Equivalent to 2-separate models

Defining groups



Maybe two groups is too simplistic

We could model **3 groups** ...

or **10 groups** ...

or **100 groups** ...

or **each person** has is their **own group** (individual based model)



Maybe two groups is too simplistic

More groups = more computationally expensive & harder to parameterize

IBMs amplify this effect



Key questions for compartmental models

Number of groups?

Size of the groups?

Differences between the groups i.e., the transmission matrix?



How groups are typically defined

We can use **demographic data** such as age, race, urban/rural

Benefits: available, widely understood, clear demarcations of groups

Drawbacks: not always directly related to transmission



An alternative approach

We can combine **behavioral & serological data**

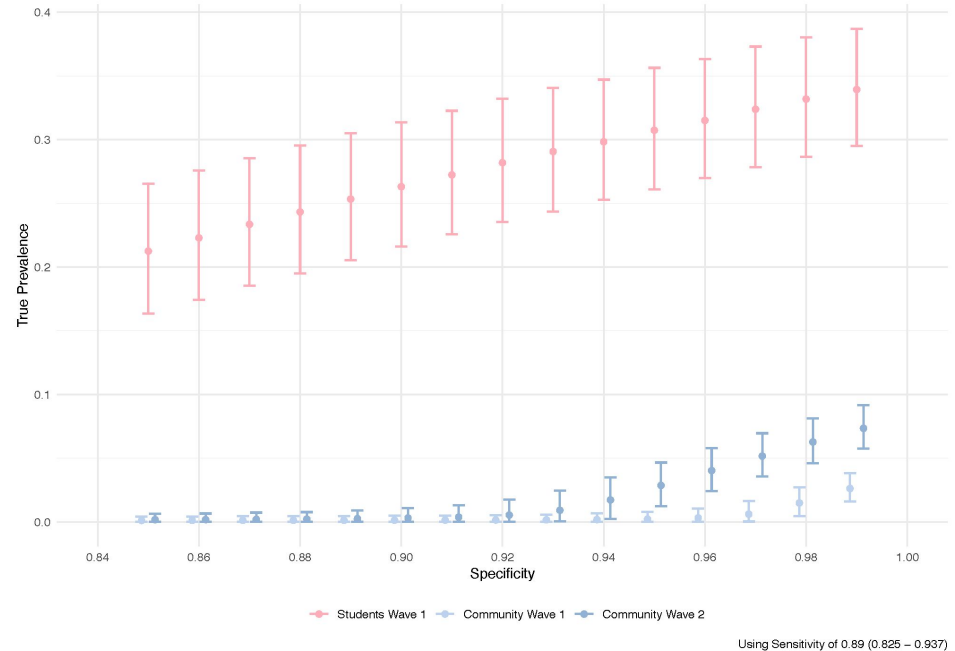
Benefits: directly related to transmission

Drawbacks: requires different data to be collected

A Latent Class Analysis (LCA) case study

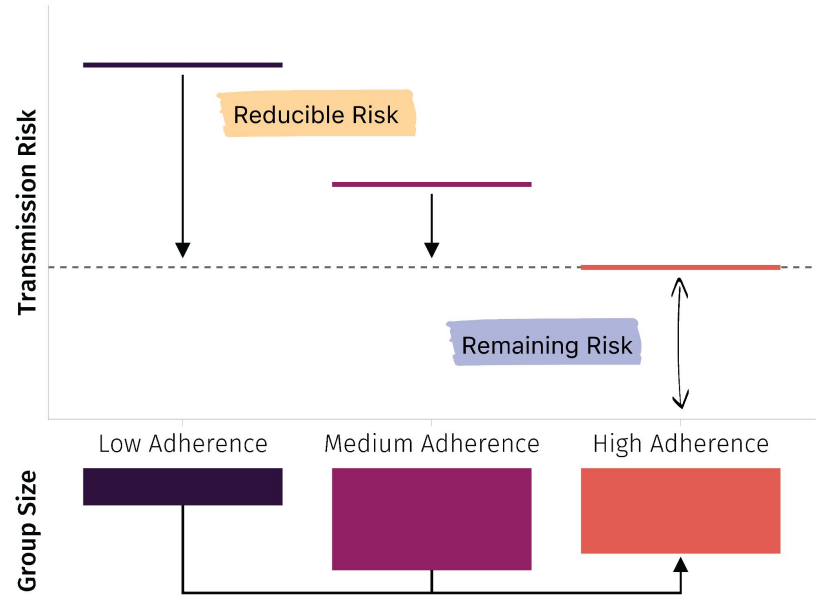
An LCA case study

- In Fall 2020, 684 students and 1313 community residents participated in a longitudinal cohort study
- High (30.4%) seroprevalence observed among the students, but low (3.2% & 7.3%) among community residents



An LCA case study

- In Fall 2020, 673 PSU students participated in **serological and behavioral surveys** about non pharmaceutical intervention (NPI) adherence for SARS-CoV-2
- Expected **heterogeneity in transmission risk**
- No obvious way to define the groups





An LCA case study

Latent-class analysis (LCA) used to **cluster students by behavioral response**

Defines:

- **Number of groups**
- **Groups sizes**

Measure Intend to always:	Low Adherence	Medium Adherence	High Adherence
Avoid face-touching with unwashed hands	0.06	0.57	0.96
Wear a mask in public	0.15	0.88	0.99
Avoid face-touching with unwashed hands	0.00	0.21	0.87
Cover cough and sneeze	0.23	0.87	1.00
Stay home when ill	0.08	0.83	1.00
Seek medical attention when have symptoms and call in advance	0.04	0.71	0.99
Stay at least 6 feet (about 2 arms lengths) from other people when outside of my home.	0.00	0.20	0.88
Stay out of crowded places and avoid mass gatherings > 25 people	0.04	0.40	0.88
GROUP SIZE	16.50%	45.50%	38.00%



An LCA case study

Use **seroprevalence** results to define **transmission risk** in each group

Key points:

- Only **62% can have their risk lowered**
- **Risk >> 0** even in most adherent group

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Stay out of crowded places and avoid mass gatherings > 25 people	0.04	0.40	0.88
GROUP SIZE	16.50%	45.50%	38.00%
SEROPREVALENCE	37.80%	32.00%	25.40%

An LCA case study

Use SIR model to explore the effect of moving individuals into high adherence group

A fully **effective** intervention results in **c. 60% reduction** in transmission

